DSC 140B - Quiz 05

February 12, 2026

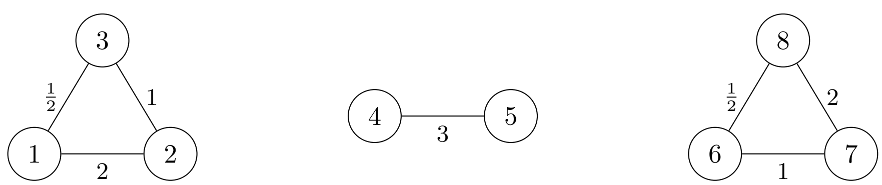
Name: PID:

About the quizzes:

- Quizzes in DSC 140B are *optional* and graded pass/fail.
- A score of 70% or higher earns a "pass" and 1.5 credits toward your final grade.
- If you don't pass, no credits are earned, but it doesn't hurt your grade.
- You have 30 minutes to complete the quiz.
- At least one of the questions below will be on an exam (probably with slight changes, such as different numbers).
- Unfortunately, we can't answer clarifying questions during the quiz. If you think a question has a bug or is unclear, please let us known in a private post on Campuswire after the quiz, and we'll take it into account when grading.

Problem 1.

Consider the similarity graph shown below with three connected components:



The edge weights are shown on each edge. Consider the embedding vector:

$$\vec{f} = (3, 3, 3, -1, -1, 5, 5, 5)^T$$

What is the Laplacian Eigenmaps cost of this embedding?

Problem 2.

Suppose you are given the following basis functions that define a feature map $\vec{\phi} : \mathbb{R}^3 \to \mathbb{R}^4$:

$$\begin{aligned}
\varphi_1(\vec{x}) &= x_1^2 \\
\varphi_2(\vec{x}) &= x_2 x_3 \\
\varphi_3(\vec{x}) &= x_1 x_3 \\
\varphi_4(\vec{x}) &= x_1 x_2
\end{aligned}$$

What is the representation of the data point $\vec{x} = (2, -1, 3)$ in the new feature space?

- ◯ $(4, 3, 6, -2)$
- ◯ $(4, -3, -6, -2)$
- ◯ $(4, -3, 6, -2)$
- ◯ $(2, -3, 6, -2)$

Problem 3.

Suppose we have a feature map $\varphi : \mathbb{R}^3 \to \mathbb{R}^4$ with the following basis functions:

$$\begin{aligned}
\varphi_1(\vec{x}) &= |x_1 + x_2| \\
\varphi_2(\vec{x}) &= |x_1 - x_3| \\
\varphi_3(\vec{x}) &= |x_2| \\
\varphi_4(\vec{x}) &= |x_2 + x_3|
\end{aligned}$$

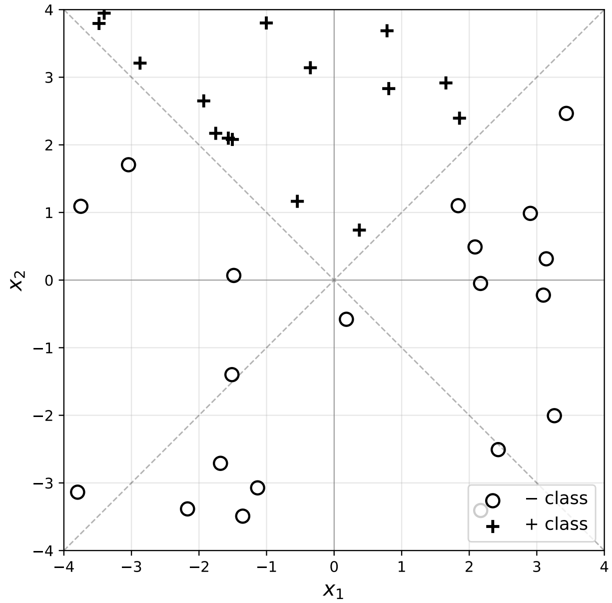
A linear prediction function in this feature space has learned the weight vector $\vec{w} = (w_0, w_1, w_2, w_3, w_4) = (1,\ 2,\ -3,\ 1,\ -1)$, where $w_0 = 1$ is the bias (intercept) term. The prediction function is:

$$H(\vec{x}) = w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x}) + w_3\varphi_3(\vec{x}) + w_4\varphi_4(\vec{x})$$

What is the value of $H(\vec{x})$ for the input point $\vec{x} = (2, -4, 1)$?

Problem 4.

Consider the data shown below:



The data comes from two classes: $\circ$ (negative) and $+$ (positive). The dashed lines represent $y = x$ and $y = -x$.

Suppose a single basis function will be used to map the data to a 1-dimensional feature space where a linear classifier will be trained. Which of the below is the best choice of basis function?

- ◯ $\varphi(x_1, x_2) = x_1 + x_2$
- ◯ $\varphi(x_1, x_2) = x_1 \cdot x_2$
- ◯ $\varphi(x_1, x_2) = x_2 - |x_1|$
- ◯ $\varphi(x_1, x_2) = x_1^2 + x_2^2$

Problem 5. (3 points)

Define the "box" basis function:

$$\phi(x;c) = \begin{cases} 1, & |x-c| \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Three box basis functions ϕ_1, ϕ_2, ϕ_3 have centers $c_1 = 0$, $c_2 = 3$, and $c_3 = 5$, respectively. These basis functions map data from $\mathbb{R}$ to feature space $\mathbb{R}^3$ via $x \mapsto (\phi_1(x), \phi_2(x), \phi_3(x))^T$.

A linear predictor in feature space has equation:

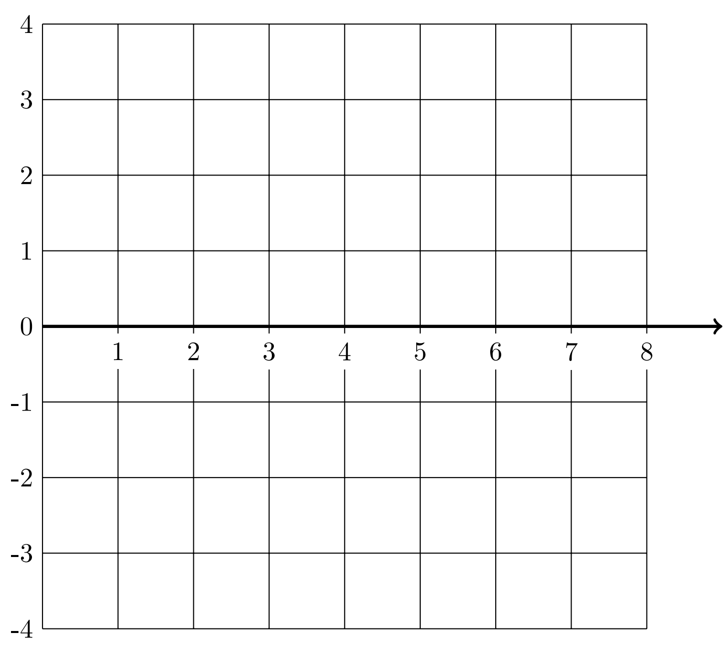
$$H_\phi(\vec{z}) = 4z_1 + 2z_2 - 3z_3$$

a) What is the representation of $x = 4$ in feature space?

- ○ $(1,1,0)^T$
- ○ $(0,1,1)^T$
- ○ $(0,1,0)^T$
- ○ $(1,0,1)^T$

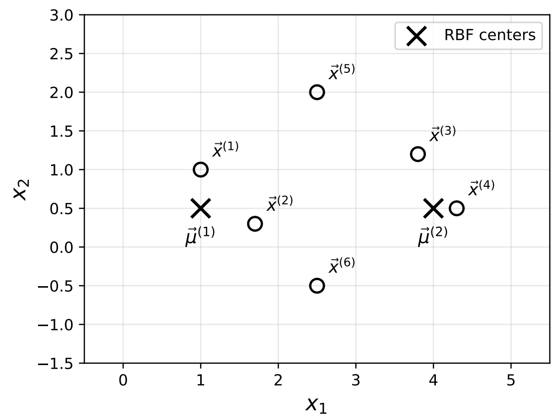
b) What is $H(2)$?

c) Plot $H(x)$ (the prediction function in the original space) from 0 to 8 on the grid below.

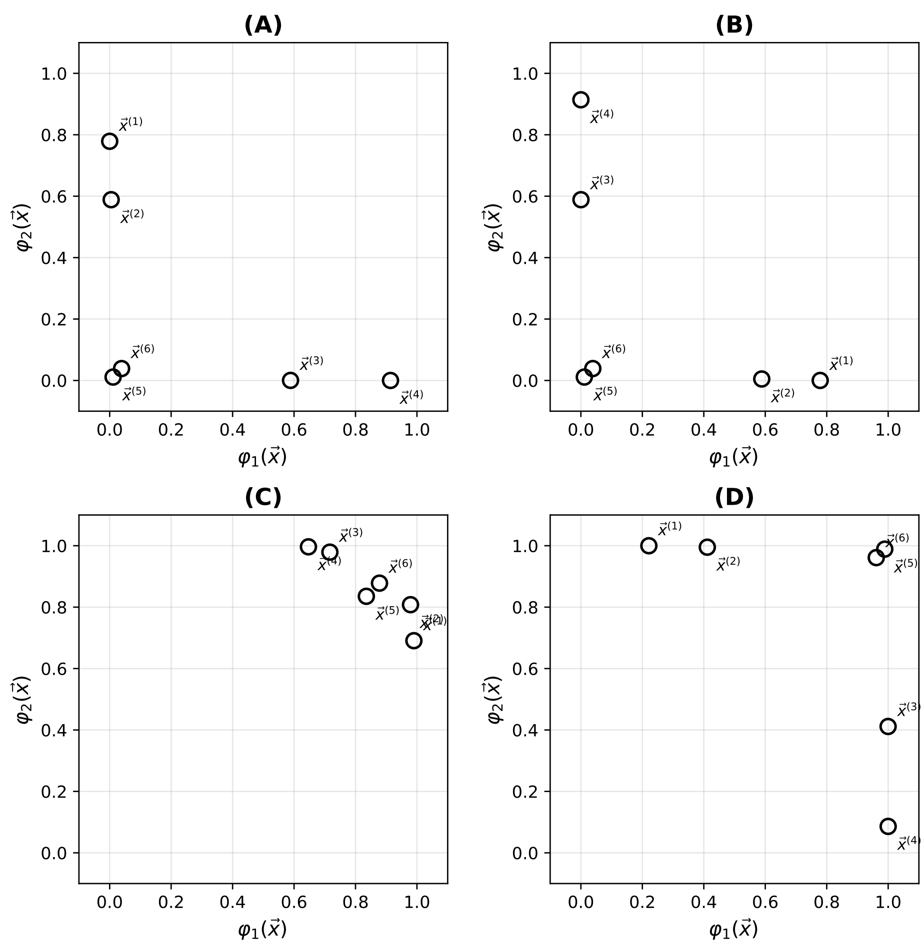


Problem 6.

Consider 6 data points $\vec{x}^{(1)}, \ldots, \vec{x}^{(6)}$ shown below in the original feature space, along with two Gaussian RBF basis function centers $\vec{\mu}^{(1)}$ and $\vec{\mu}^{(2)}$ (shown as x markers). Both RBF basis functions use $\sigma = 1$.



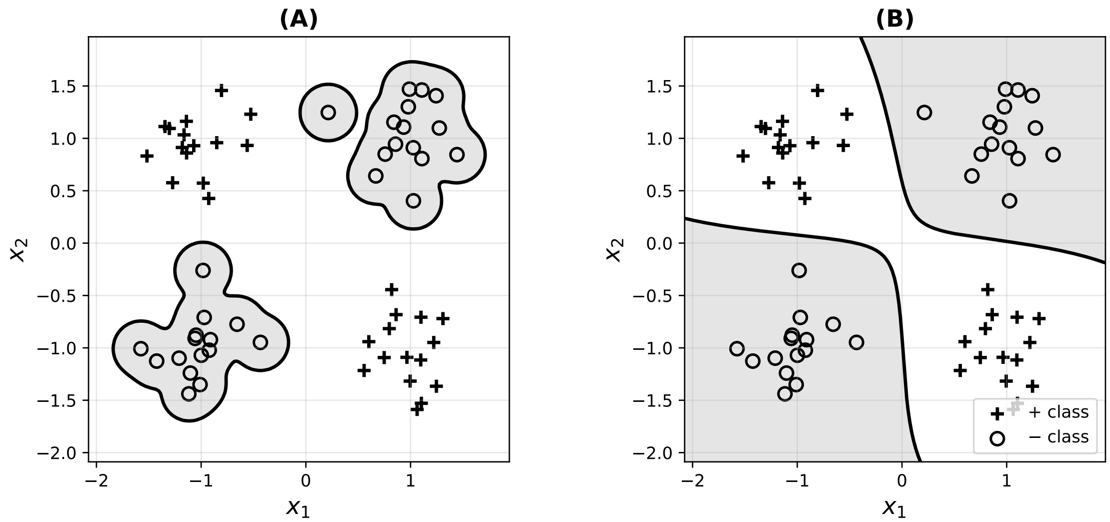
One of the plots below shows the data after it has been mapped to feature space using the two Gaussian RBF basis functions centered at $\vec{\mu}^{(1)}$ and $\vec{\mu}^{(2)}$. Which plot is it?



◯ (A) ◯ (B) ◯ (C) ◯ (D)

Problem 7.

Below are two decision boundaries produced by Gaussian RBF classifiers trained on the same data. Both classifiers use the same RBF basis function centers, but with different width parameters σ.



Which classifier uses a larger value of σ?

○ (A)

○ (B)

Problem 8.

Does standardizing features affect the output of a Gaussian RBF network?

More precisely, let $\mathcal{X} = \{(\vec{x}^{(1)}, y_1), \ldots, (\vec{x}^{(n)}, y_n)\}$ be a dataset, and let $\mathcal{Z} = \{(\vec{z}^{(1)}, y_1), \ldots, (\vec{z}^{(n)}, y_n)\}$ be the dataset obtained by standardizing each feature in $\mathcal{X}$ (that is, each feature is scaled to have unit standard deviation and zero mean). Note that the y_i's are not standardized – they are the same in both datasets.

Suppose $H_{\mathcal{X}}(\vec{x})$ is a Gaussian RBF network trained by minimizing mean squared error on $\mathcal{X}$, and $H_{\mathcal{Z}}(\vec{z})$ is a Gaussian RBF network trained by minimizing mean squared error on $\mathcal{Z}$. Both RBF networks use the same width parameters, σ. The centers used in $H_{\mathcal{Z}}$ are obtained by standardizing the centers used in $H_{\mathcal{X}}$ using the mean and standard deviation of the data, $\vec{x}^{(i)}$.

True or False: it must be the case that $H_{\mathcal{X}}(\vec{x}^{(i)}) = H_{\mathcal{Z}}(\vec{z}^{(i)})$ for all $i \in \{1, \ldots, n\}$.

○ True

○ False