DSC 140B - Quiz 03

Name: PID:

About the quizzes:

- Quizzes in DSC 140B are *optional* and graded pass/fail.
- A score of 70% or higher earns a "pass" and 1.5 credits toward your final grade.
- If you don't pass, no credits are earned, but it doesn't hurt your grade.
- You have 30 minutes to complete the quiz.
- At least one of the questions below will be on an exam (probably with slight changes, such as different numbers).
- Unfortunately, we can't answer clarifying questions during the quiz. If you think a question has a bug or is unclear, please let us known in a private post on Campuswire after the quiz, and we'll take it into account when grading.

Problem 1.

Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ and let $\vec{v} = (2, -1, -1)^T$.

Is $\vec{v}$ an eigenvector of A?

◯ Yes, with eigenvalue $\lambda = 2$

◯ Yes, with eigenvalue $\lambda = 3$

◯ Yes, with eigenvalue $\lambda = 4$

◯ Yes, with eigenvalue $\lambda = 5$

◯ No, $\vec{v}$ is not an eigenvector of A

Problem 2.

Let A be a matrix with eigenvector $\vec{v}$ and corresponding eigenvalue -2.

True or False: $\vec{v}$ is an eigenvector of A^2 with eigenvalue 4.

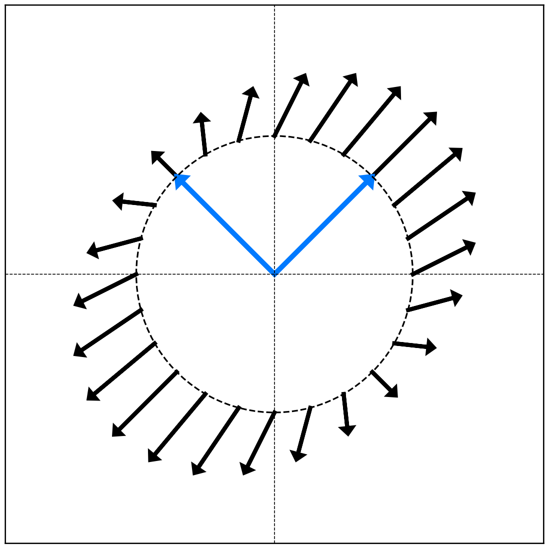
(Remember, $A^2 = AA$; that is, it is the matrix A multiplied by itself.)

◯ True

◯ False

Problem 3. (2 points)

The figure below shows a linear transformation $\vec{f}$ applied to points on the unit circle. Each arrow shows the direction and relative magnitude of $\vec{f}(\vec{x})$ for a point $\vec{x}$ on the circle.



The eigenvectors of the matrix A of this transformation are $\hat{u}^{(1)} = \frac{1}{\sqrt{2}}(1,1)^T$ and $\hat{u}^{(2)} = \frac{1}{\sqrt{2}}(1,-1)^T$. They are also plotted here as two arrows landing on the unit circle.

a) True or False: The matrix A is symmetric.

- ◯ True
- ◯ False

b) True or False: The matrix A is diagonal.

- ◯ True
- ◯ False

c) Which eigenvector corresponds to the larger eigenvalue?

- ◯ $\hat{u}^{(1)} = \frac{1}{\sqrt{2}}(1,1)^T$
- ◯ $\hat{u}^{(2)} = \frac{1}{\sqrt{2}}(1,-1)^T$

Problem 4.

Let $\hat{u}^{(1)}, \hat{u}^{(2)}, \hat{u}^{(3)}$ be orthonormal eigenvectors of a linear transformation $\vec{f}$, with eigenvalues 3, -1, and 2 respectively.

Suppose $\vec{x} = 3\hat{u}^{(1)} - 2\hat{u}^{(2)} + 4\hat{u}^{(3)}$.

What is $\vec{f}(\vec{x})$ in coordinates with respect to the eigenbasis $\mathcal{U} = \{\hat{u}^{(1)}, \hat{u}^{(2)}, \hat{u}^{(3)}\}$? That is, what is $[\vec{f}(\vec{x})]_{\mathcal{U}}$?

- ◯ $(9,-2,8)^T$
- ◯ $(9,2,8)^T$
- ◯ $(3,-2,4)^T$
- ◯ $(9,2,-8)^T$

Problem 5.

Let $\vec{f}$ be a symmetric linear transformation in $\mathbb{R}^3$ with orthonormal eigenvectors $\hat{u}^{(1)}$, $\hat{u}^{(2)}$, $\hat{u}^{(3)}$ and corresponding eigenvalues 4, −7, and 2.

What is the maximum value of $\|\vec{f}(\vec{x})\|$ over all unit vectors $\vec{x}$?

Problem 6.

Let $\hat{u}^{(1)}$ and $\hat{u}^{(2)}$ be an orthonormal basis for $\mathbb{R}^2$:

$$\hat{u}^{(1)} = \frac{1}{\sqrt{2}}(1,1)^T$$
$$\hat{u}^{(2)} = \frac{1}{\sqrt{2}}(-1,1)^T$$

Let $\vec{x} = (1,3)^T$ be a vector in the standard basis. What are the coordinates of $\vec{x}$ in the basis $\mathcal{U} = \{\hat{u}^{(1)}, \hat{u}^{(2)}\}$? That is, what is $[\vec{x}]_{\mathcal{U}}$?

- ◯ $(\sqrt{2}, 2\sqrt{2})^T$
- ◯ $(2,1)^T$
- ◯ $(2\sqrt{2}, \sqrt{2})^T$
- ◯ $(4,2)^T$

Problem 7.

Let $\vec{f}$ be a linear transformation with eigenvectors and eigenvalues:

$$\hat{u}^{(1)} = \frac{1}{\sqrt{2}}(1,1)^T \qquad \lambda_1 = 3$$
$$\hat{u}^{(2)} = \frac{1}{\sqrt{2}}(1,-1)^T \qquad \lambda_2 = -1$$

What is $\vec{f}(\vec{x})$ for $\vec{x} = (2,0)^T$?

- ◯ $(4,2)^T$
- ◯ $(6,-2)^T$
- ◯ $(3,-1)^T$
- ◯ $(2,4)^T$

Problem 8.

Consider the matrix $A = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$.

The three eigenvectors of A are:

$$\begin{aligned} \hat{u}^{(1)} &= \frac{1}{\sqrt{3}}(1,1,1)^T \\ \hat{u}^{(2)} &= \frac{1}{\sqrt{2}}(1,-1,0)^T \\ \hat{u}^{(3)} &= \frac{1}{\sqrt{6}}(1,1,-2)^T \end{aligned}$$

What does the matrix A look like in the eigenbasis $\mathcal{U} = \{\hat{u}^{(1)}, \hat{u}^{(2)}, \hat{u}^{(3)}\}$?

- ○ $\begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$
- ○ $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$
- ○ $\begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$
- ○ $\begin{pmatrix} 6 & 3 & 3 \\ 3 & 6 & 3 \\ 3 & 3 & 6 \end{pmatrix}$