DSC 140B

Representation Learning

Lecture 10 | Part 1

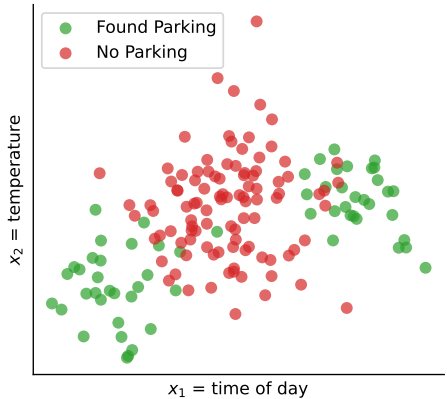
Recap: Feature Maps

Problem

- Linear predictors only learn **linear** patterns.
  - Straight lines / planes.

- Patterns in real world data are often **non-linear**.

Example: Classification



Idea

- The pattern is **non-linear** in its original representation.

- **Idea:**
  1. Change to a new representation where the pattern is **linear**.

  2. Train a linear predictor in the new representation.

Idea

- To change representation, we use non-linear **feature maps**.

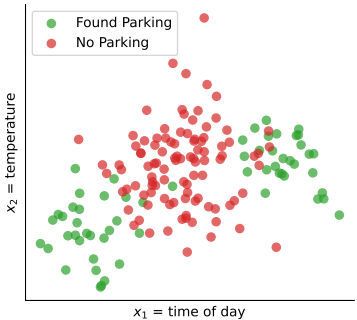
$$\vec{\phi}(\vec{x}) = (\vec{\phi}_1(\vec{x}), \vec{\phi}_2(\vec{x}), \dots, \vec{\phi}_k(\vec{x}))^T$$

- A (good) feature map can turn **non-linear** patterns into **linear** patterns.

Procedure: Learning with Feature Maps

- First, pick a feature map $\vec{\phi} : \mathbb{R}^d \to \mathbb{R}^k$.
- **To train**:
  - Given training set $(\vec{x}^{(1)}, y_1), \ldots, (\vec{x}^{(n)}, y_n)$.
  1. Map each $\vec{x}^{(i)}$ to feature space, creating a new data set $(\vec{\phi}(\vec{x}^{(1)}), y_1), \ldots, (\vec{\phi}(\vec{x}^{(n)}), y_n)$.
  2. Train linear model on the new data in feature space to get $\vec{w}^*$.
- **To predict:**
  - Given new input $\vec{x}$.
  1. Map $\vec{x}$ to feature space: $\vec{\phi}(\vec{x})$.
  2. Predict $H(\vec{x}; \vec{w}^*) = \vec{w}^* \cdot \text{Aug}(\vec{\phi}(\vec{x}))$.

Example: Parking Prediction



- Original features:

$$\vec{x} = (\text{time}, \text{temp.})^T$$

- Feature map:

$$\vec{\phi}(\vec{x}) = (|\text{time} - \text{Noon}|, |\text{temp.} - 70|)^T$$

- We'll train a **least squares classifier** in feature space.

Step 1: Pick a Feature Map

- In the input space, we have features (x_1, x_2).

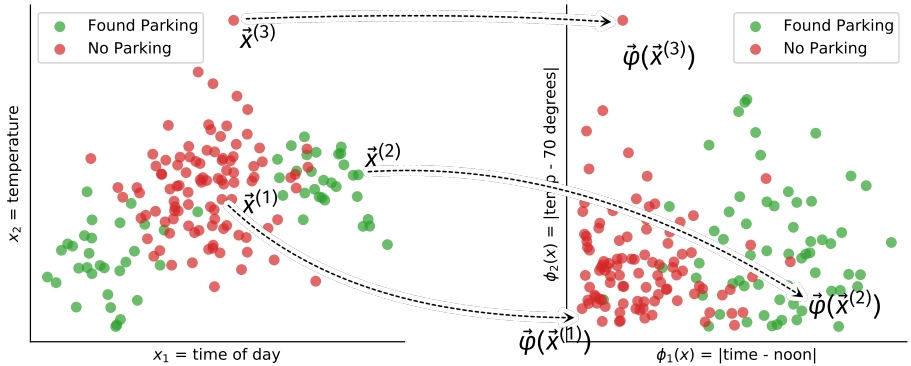
$$x_1 = \text{time}, \quad x_2 = \text{temperature}.$$

- We'll use the same feature map as before:

$$\vec{\phi}(x_1, x_2) = (|x_1 - 12|, |x_2 - 70|)^T$$

Step 2(a): Map to Feature Space

- Map every data point to feature space.



Step 2(b): Train in Feature Space

- In feature space, our feature vectors are $\vec{\phi}(\vec{x}^{(1)}), \ldots, \vec{\phi}(\vec{x}^{(n)})$.

- So the design matrix becomes the $(n \times k)$ matrix:

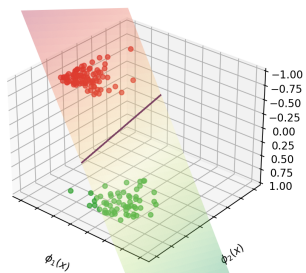
$$\Phi = \begin{pmatrix} \text{Aug}(\vec{\phi}(\vec{x}^{(1)})) \longrightarrow \\ \text{Aug}(\vec{\phi}(\vec{x}^{(2)})) \longrightarrow \\ \vdots \\ \text{Aug}(\vec{\phi}(\vec{x}^{(n)})) \longrightarrow \end{pmatrix} = \begin{pmatrix} 1 & |x_1^{(1)} - 12| & |x_2^{(1)} - 70| \\ 1 & |x_1^{(2)} - 12| & |x_2^{(2)} - 70| \\ \vdots & \vdots & \vdots \\ 1 & |x_1^{(n)} - 12| & |x_2^{(n)} - 70| \end{pmatrix}$$

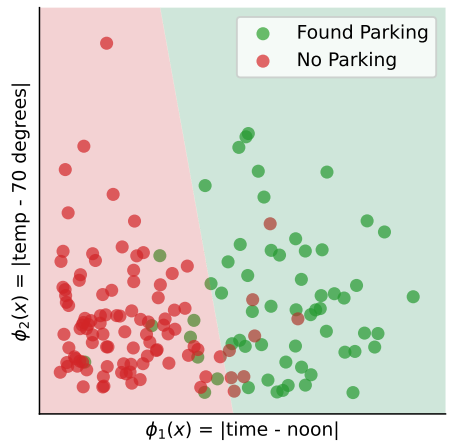
Step 2(b): Train in Feature Space

- The least squares solution in feature space is:

$$\vec{w}^* = (\Phi^T \Phi)^{-1} \Phi^T \vec{y}$$

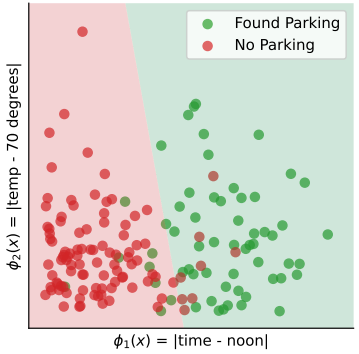
Solution in Feature Space





Step 3: Predict

- Given a new example $\vec{x}$ in input space:
  1. Map $\vec{x}$ to feature space: $\vec{\phi}(\vec{x})$.
  2. Predict sign($\vec{w}^* \cdot \text{Aug}(\vec{\phi}(\vec{x}))$).



Exercise

Let $\vec{\phi}(x_1, x_2) = (|x_1 - 12|, |x_2 - 70|)^T$. Suppose we train a least squares classifier in feature space and find $\vec{w}^* = (3, -1, 2)^T$.

Given a new point $\vec{x} = (10, 65)^T$ in input space, what is the prediction, $H(\vec{x})$?

The Prediction Function(s)

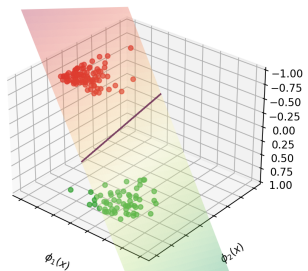
- There are, in a sense, *two* prediction functions:
  - H, which accepts inputs in **input** space.
  - H_ϕ, which accepts inputs in **feature** space.

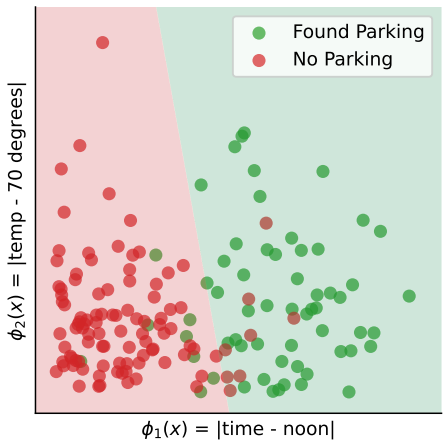
- First, the prediction function in **feature space**:

$$H_\phi(\vec{z}) = \vec{w} \cdot \text{Aug}(\vec{z})$$
$$= w_0 + w_1 z_1 + w_2 z_2 + \ldots + w_k z_k$$

H_ϕ in Feature Space

$$H_\phi(\vec{z}) = w_0 + w_1 z_1 + w_2 z_2$$





The Prediction Function

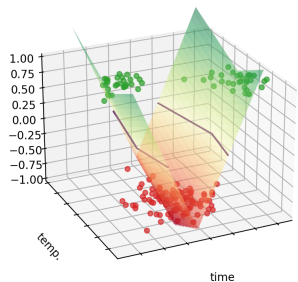
- There is also the prediction function $H(\vec{x})$ that takes in vectors in **input space**.

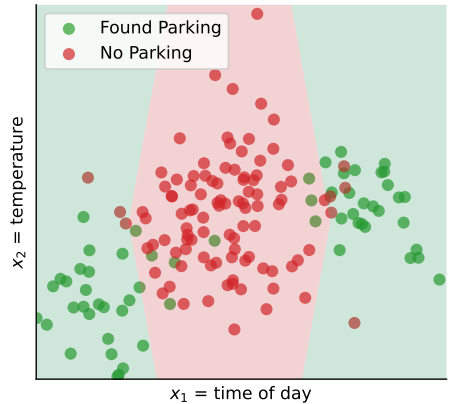
$$\begin{aligned} H(\vec{x}) &= H_{\phi}(\vec{\phi}(\vec{x})) \\ &= \vec{w} \cdot \text{Aug}(\vec{\phi}(\vec{x})) \\ &= w_0 + w_1\phi_1(\vec{x}) + w_2\phi_2(\vec{x}) + \ldots + w_k\phi_k(\vec{x}) \end{aligned}$$

- When plotted, this function will look **non-linear**.

H in Input Space

$$H(\vec{x}) = w_0 + w_1|x_1 - 12| + w_2|x_2 - 70|$$





Exercise

Let $\vec{\phi}(x_1, x_2) = (|x_1 - 12|, |x_2 - 70|)^T$. Suppose we train a least squares classifier in feature space and find $\vec{w}^* = (3, -1, 2)^T$.

Given a new point $\vec{x} = (10, 65)^T$ in input space, what is the prediction, $H(\vec{x})$? This time, compute the answer *without* explicitly computing $\vec{\phi}(\vec{x})$.

Problem

- Feature maps can learn non-linear patterns.
- But so far, we've found them **manually**.
- How do we find a good one **automatically**?

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Representation Learning

Lecture 10 | Part 2

Radial Basis Functions

Overview: Feature Mapping

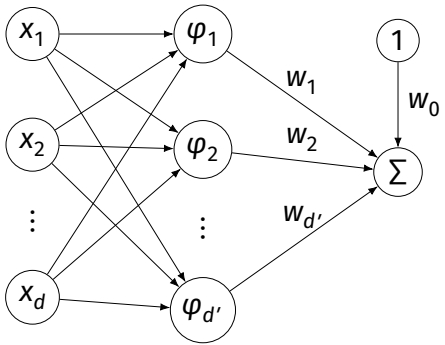
1. Start with data in original space, $\mathbb{R}^d$.
2. Choose some basis functions, $\varphi_1, \varphi_2, \ldots, \varphi_{d'}$
3. Map each data point to **feature space** $\mathbb{R}^{d'}$:

$$\vec{x} \mapsto (\varphi_1(\vec{x}), \varphi_2(\vec{x}), \ldots, \varphi_{d'}(\vec{x}))^t$$

4. Fit linear prediction function in new space:

$$H(\vec{x}) = w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x})$$

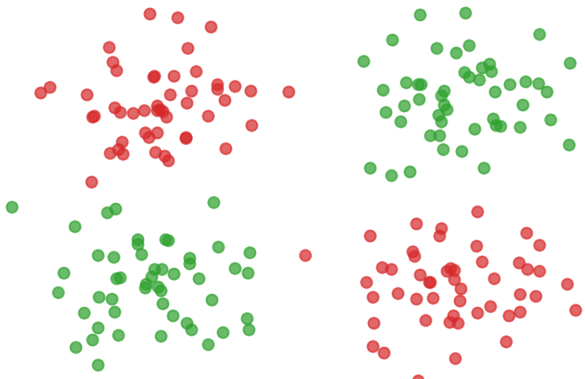
$$H(\vec{x}) = w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x})$$



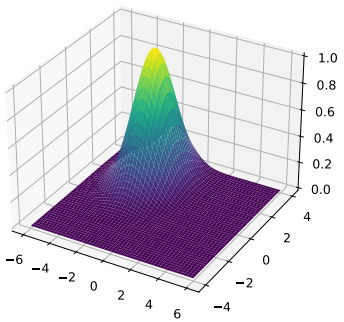
Generic Basis Functions

- The basis functions we used before were engineered using domain knowledge.
- They were specific to the problem at hand.
- **Very manual process!**
- **Now:** features that work for many problems.

Example



Gaussian Basis Functions



- A common choice: **Gaussian** basis functions:

$$\varphi(\vec{x}; \vec{\mu}, \sigma) = e^{-\|\vec{x}-\vec{\mu}\|^2/\sigma^2}$$

- $\vec{\mu}$ is the center.
- σ controls the “width”

Gaussian Basis Function

- If $\vec{x}$ is close to $\vec{\mu}$, $\varphi(\vec{x}; \vec{\mu}, \sigma)$ is large.
- If $\vec{x}$ is far from $\vec{\mu}$, $\varphi(\vec{x}; \vec{\mu}, \sigma)$ is small.
- Intuition: φ measures how "similar" $\vec{x}$ is to $\vec{\mu}$.
  - Assumes that "similar" objects have close feature vectors.

New Representation

- Pick number of new features, d'.
- Pick centers for Gaussians $\vec{\mu}^{(1)}, \ldots, \vec{\mu}^{(2)}, \ldots, \vec{\mu}^{(d')}$
- Pick widths: $\sigma_1, \sigma_2, \ldots, \sigma_{d'}$ (usually all the same)
- Define ith basis function:

$$\varphi_i(\vec{x}) = e^{-\|\vec{x}-\vec{\mu}^{(i)}\|^2/\sigma_i^2}$$

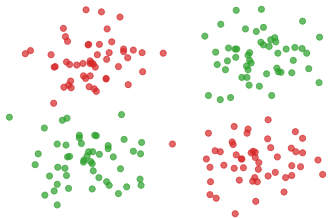
New Representation

- For any feature vector $\vec{x} \in \mathbb{R}^d$, map to vector $\vec{\varphi}(\vec{x}) \in \mathbb{R}^{d'}$.
  - φ_1: "similarity" of $\vec{x}$ to $\vec{\mu}^{(1)}$
  - φ_2: "similarity" of $\vec{x}$ to $\vec{\mu}^{(2)}$
  - ...
  - $\varphi_{d'}$: "similarity" of $\vec{x}$ to $\vec{\mu}^{(d')}$

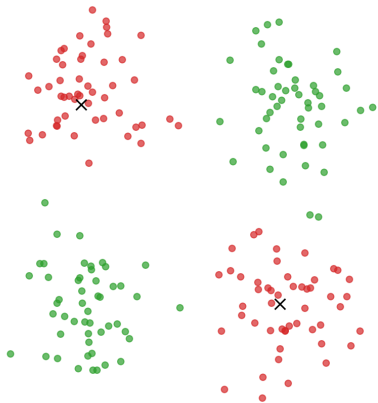
- Train linear classifier in this new representation.
  - E.g., by minimizing expected square loss.

Exercise

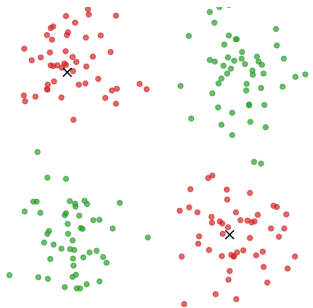
How many Gaussian basis functions would you use, and where would you place them to create a new representation for this data?

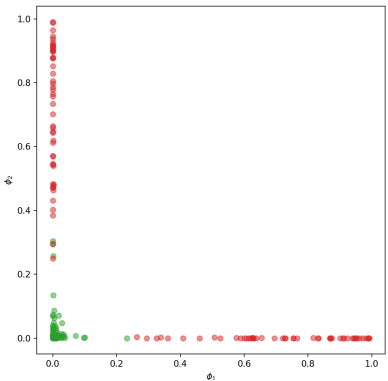


Placement



Feature Space





Prediction Function

- $H(\vec{x})$ is a sum of Gaussians:

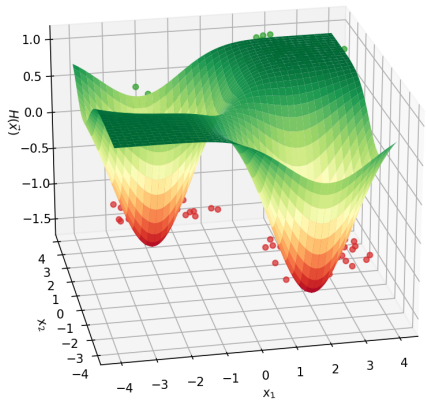
$$H(\vec{x}) = w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x}) + \ldots$$
$$= w_0 + w_1 e^{-\|\vec{x}-\vec{\mu}_1\|^2/\sigma^2} + w_2 e^{-\|\vec{x}-\vec{\mu}_2\|^2/\sigma^2} + \ldots$$

Exercise

What does the surface of the prediction function look like?

Hint: what does the sum of 1-d Gaussians look like?

Prediction Function Surface

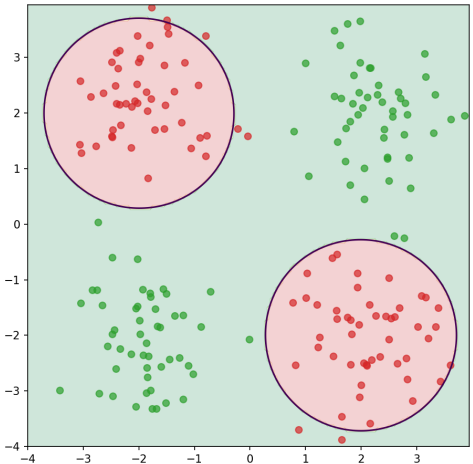


$$H(\vec{x}) = w_0 + w_1 e^{-\|\vec{x}-\vec{\mu}_1\|^2/\sigma^2} + w_2 e^{-\|\vec{x}-\vec{\mu}_2\|^2/\sigma^2}$$

An Interpretation

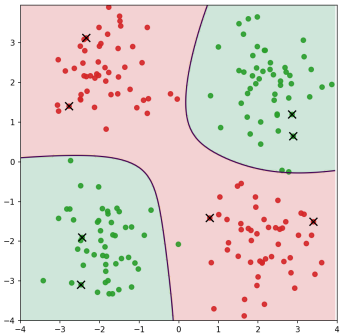
- Basis function φ_i makes a “bump” in surface of H
- w_i adjusts the “prominance” of this bump

Decision Boundary

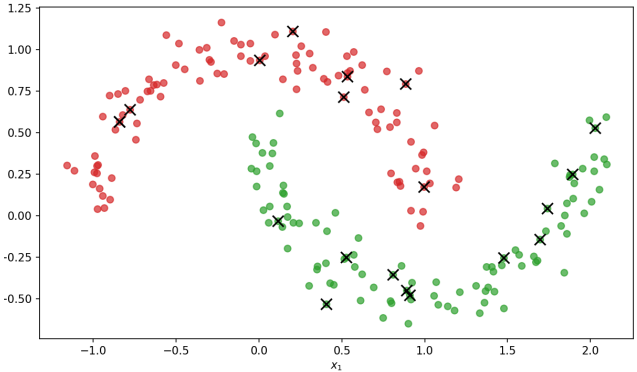


More Features

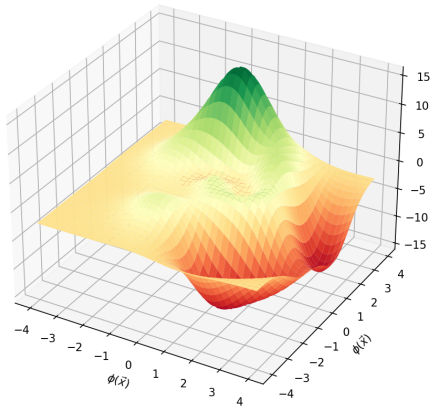
- By increasing number of basis functions, we can make more complex decision surfaces.



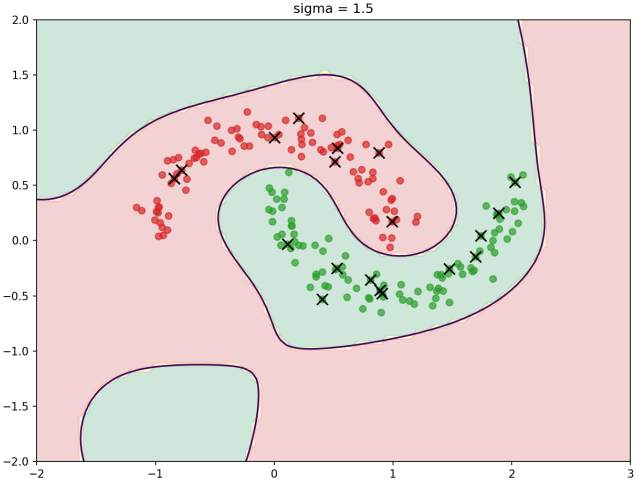
Another Example



Prediction Surface



Decision Boundary



Radial Basis Functions

- Gaussians are examples of **radial basis functions**.
- Each basis function has a **center**, $\vec{c}$.
- Value depends only on distance from center:

$$\varphi(\vec{x}; \vec{c}) = f(\|\vec{x} - \vec{c}\|)$$

Another Radial Basis Function

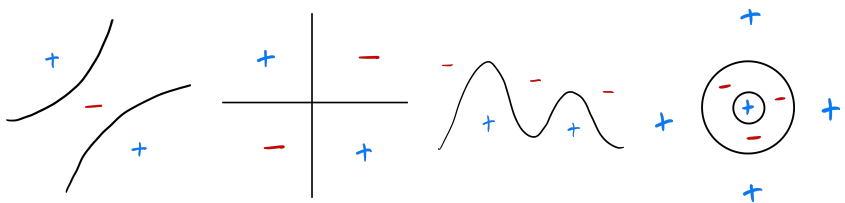
- **Multiquadric**: $\varphi(\vec{x}; \vec{c}) = \sqrt{\sigma^2 + \|\vec{x} - \vec{c}\|} / \sigma$

Exercise

Suppose we use four Gaussian RBFs (same σ) at $\vec{\mu}_1 = (0,0)$, $\vec{\mu}_2 = (1,0)$, $\vec{\mu}_3 = (0,1)$, $\vec{\mu}_4 = (1,1)$ and train a linear classifier in feature space, finding weights $w_1 = -3$, $w_2 = 3$, $w_3 = 3$, $w_4 = -3$.

Which of the below could show the decision boundary of H in **input** space?

(a) (b) (c) (d)



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Representation Learning

Lecture 10 | Part 3

Radial Basis Function Networks

Recap

1. Choose basis functions, $\varphi_1, \ldots, \varphi_{d'}$
2. Transform data to new representation:

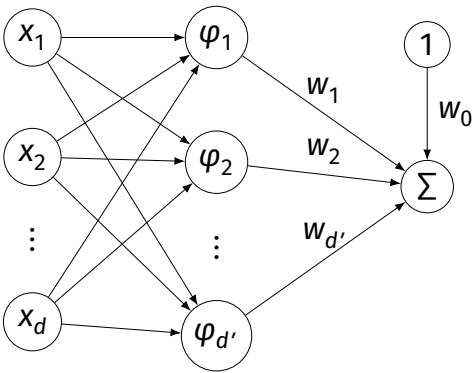
$$\vec{x} \mapsto (\varphi_1(\vec{x}), \varphi_2(\vec{x}), \ldots, \varphi_{d'}(\vec{x}))^T$$

3. Train a linear classifier in this new space:

$$H(\vec{x}) = w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x}) + \ldots + w_{d'}\varphi_{d'}(\vec{x})$$

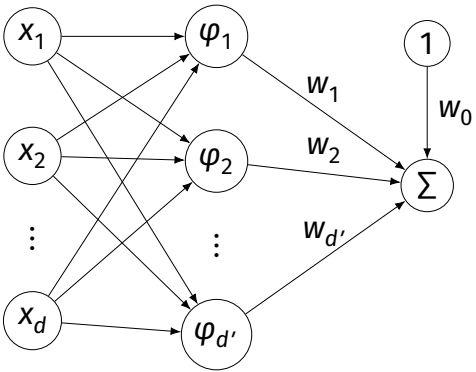
The Model

- The φ are **basis functions**.



$$H(\vec{x}) = w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x})$$

Radial Basis Function Networks



If the basis functions are **radial basis functions**, we call this a **radial basis function (RBF) network**.

Training

- An RBF network has these parameters:
  - the parameters of each individual basis function:
    - $\vec{\mu}_i$ (the center)
    - possibly others (e.g., σ)
  - w_i: the weights associated to each “new” feature

- How do we choose the parameters?

First Idea

- We can include all parameters in one big cost function, optimize.
- The cost function will generally be **complicated, non-convex** and thus **hard to optimize**.

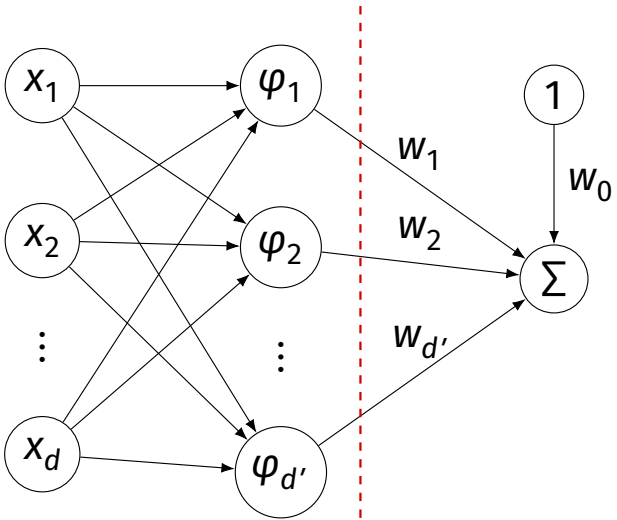
Another Idea

- Break the process into two steps:

1. Find the parameters of the RBFs *somehow*.
   - Some optimization procedure, clustering, randomly, ...
2. Having fixed those parameters, optimize the w's.

- **Linear; easier to optimize.**

Training



Training an RBF Network

1. Choose the form of the RBF, how many.
   - E.g., k Gaussian RBFs, $\varphi_1, \ldots, \varphi_k$.
2. Pick the parameters of the RBFs *somehow*.
3. Create new data set by mapping
   $\vec{x} \mapsto (\varphi_1(\vec{x}), \ldots, \varphi_k(\vec{x}))^T$
4. Train a linear predictor H_f on new data set
   - That is, in feature space.

Making Predictions

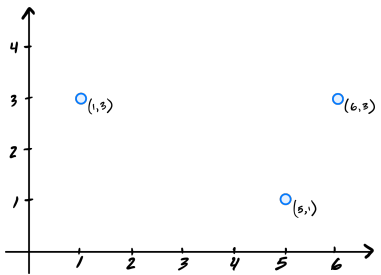
1. Given a point $\vec{x}$, map it to feature space: $\vec{x} \mapsto (\varphi_1(\vec{x}), \ldots, \varphi_k(\vec{x}))^T$

2. Evaluate the trained linear predictor H_f in feature space

Exercise

A Gaussian RBF network $H(x) = w_1\varphi_1(x) + w_2\varphi_2(x)$ (no bias) is trained on the data below using two Gaussian basis functions with centers $\mu_1 = 2$, $\mu_2 = 6$ and width $\sigma = 1$.

What is w_1, approximately?



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Representation Learning

Lecture 10 | Part 4

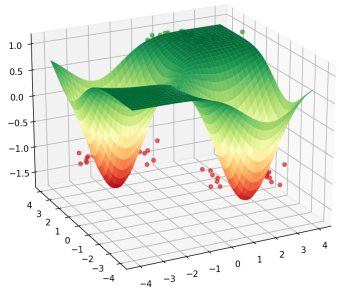
Choosing RBF Locations

Recap

- We map data to a new representation by first choosing **basis functions**.
- Radial Basis Functions (RBFs), such as Gaussians, are a popular choice.
- Requires choosing **center** for each basis function.

Prediction Function

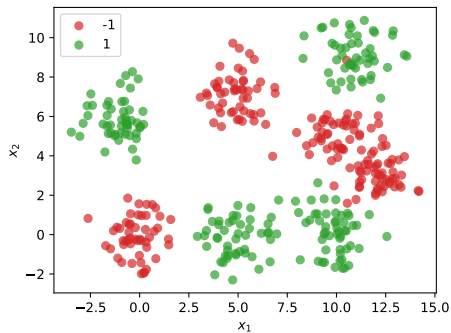
- Our prediction function H is a surface that is made up of Gaussian “bumps”.



$$H(\vec{x}) = w_0 + w_1 e^{-\|\vec{x}-\vec{\mu}_1\|^2/\sigma^2} + w_2 e^{-\|\vec{x}-\vec{\mu}_2\|^2/\sigma^2}$$

Choosing Centers

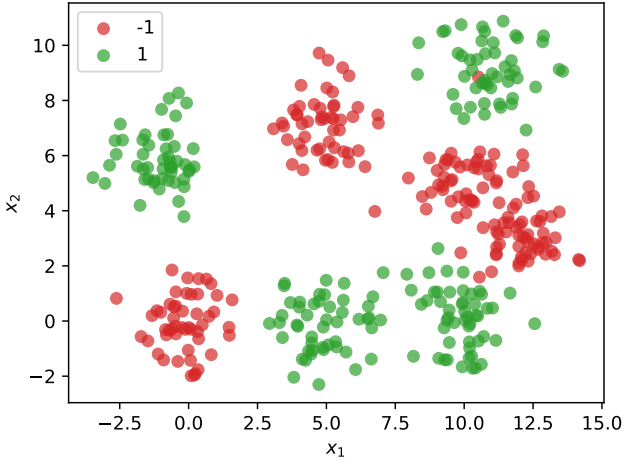
- Place the centers where the value of the prediction function should be controlled.
- Intuitively: place centers where the data is.



Approaches

1. Every data point as a center
2. Randomly choose centers
3. Clustering

Approach #1: Every Data Point as a Center



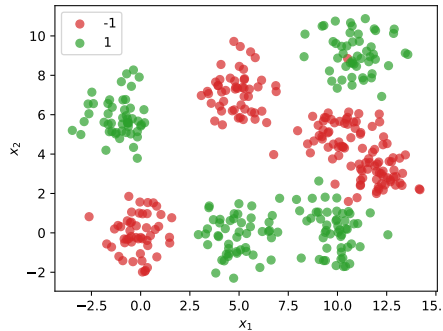
Dimensionality

- We'll have n basis functions – one for each point.
- That means we'll have n features.
- Each feature vector $\vec{\phi}(\vec{x}) \in \mathbb{R}^n$.

$$\vec{\phi}(\vec{x}) = (\phi_1(\vec{x}), \phi_2(\vec{x}), \ldots, \phi_n(\vec{x}))^T$$

Problems

- This causes problems.
- First: more likely to **overfit**.
- Second: computationally expensive

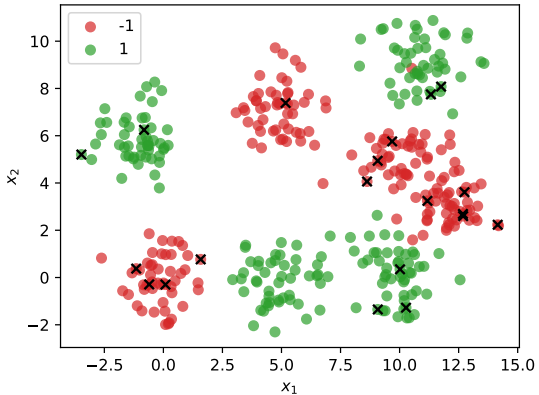


Computational Cost

- Suppose feature matrix X is $n \times d$
  - n points in d dimensions
- Time complexity of solving $X^T X\vec{w} = X^T\vec{y}$ is $\Theta(nd^2)$
- Usually $d \ll n$. But if $d = n$, this is $\Theta(n^3)$.
- Not great! If $n \approx 10,000$, then takes > 10 minutes.

Approach #2: A Random Sample

- Idea: randomly choose *k* data points as centers.



Problem

- May undersample/oversample a region.
- More advanced sampling approaches exist.

Approach #3: Clustering

- Group data points into **clusters**.
- Cluster centers are good places for RBFs.
- For example, use *k*-means clustering to pick *k* centers.

DSC 140B

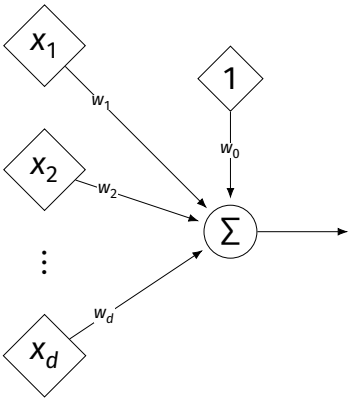
Representation Learning

Lecture 10 | Part 5

Neural Networks

Linear Models

$$H(\vec{x}) = w_0 + w_1 x_1 + \ldots + w_d x_d$$



Linear Models

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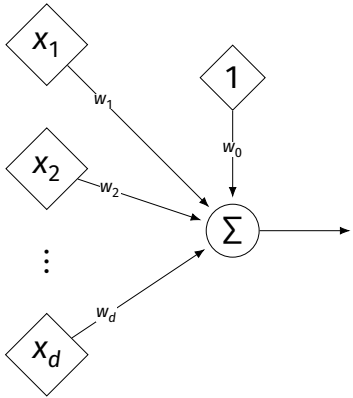
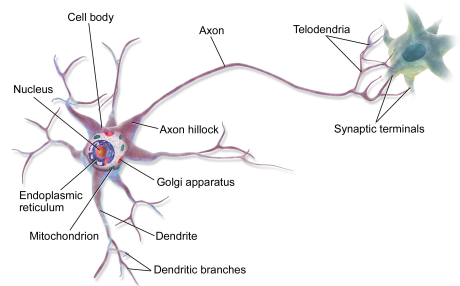
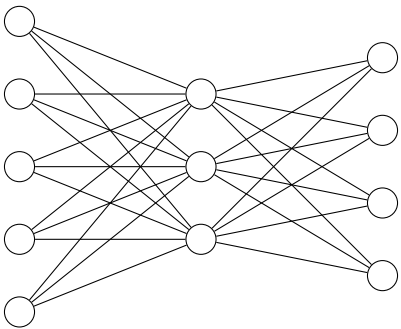




The Brain

- The brain is a **network** of neurons.
  - The **output** of a neuron is used as an **input** to others.

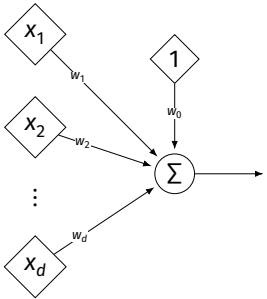


Idea: Neural Networks

- Replace brain's neurons with linear models.
- Connect them together into a **neural network**.
  - Output of one linear model is used as input to others.

A Simple Neural Network

- The simplest neural network is the one we've already seen: one neuron.



A Simple Neural Network

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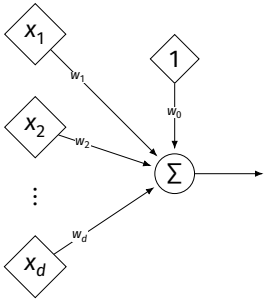
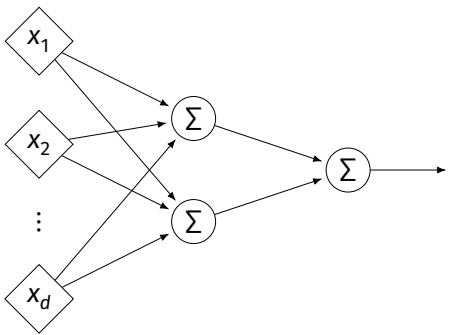


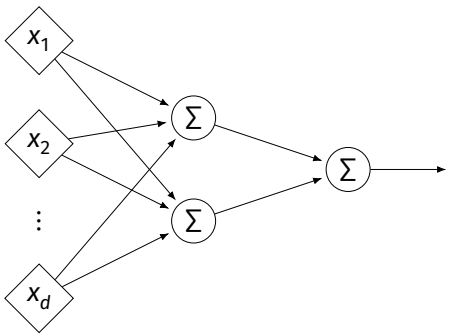
- A linear regression model is a neural network with one neuron.

Another Neural Network



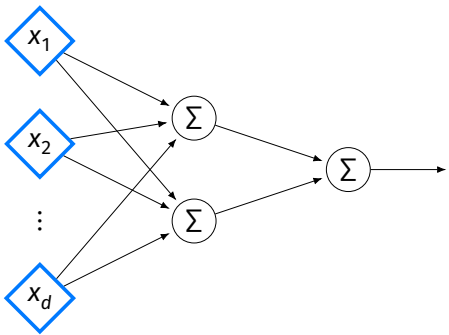
Another Neural Network

- Neural nets have **layers**.



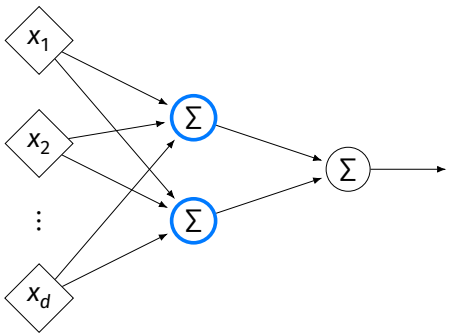
Another Neural Network

- The **input** layer (one node per feature).



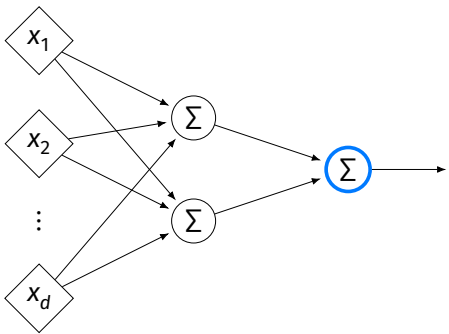
Another Neural Network

- Zero or more **hidden** layers.



Another Neural Network

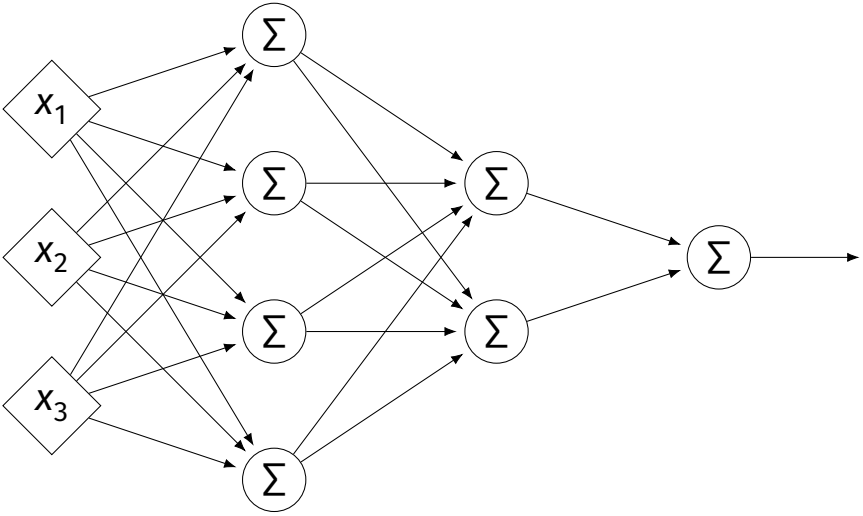
- The **output** layer (one node per output).



Architecture

- Can have more than one hidden layer.
  - A network is “**deep**” if it has >1 hidden layer.
- Hidden layers can have different number of neurons.

Neural Network (Two Hidden Layers)

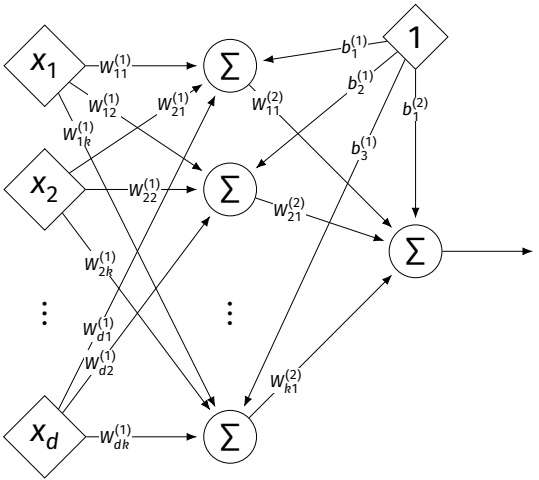


Network Weights

- Each edge in a NN has a weight that can be learned.
- Like a linear model, a NN is **totally determined** by its weights.
- But there are often many more weights to learn!

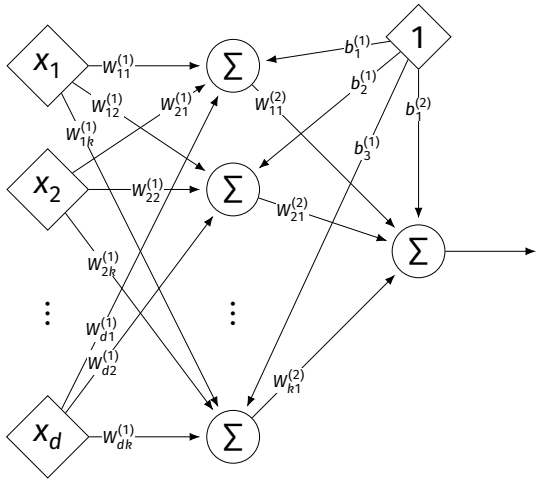
Notation

- Input is layer #0.
- $W_{jk}^{(i)}$ denotes weight of connection between neuron j in layer $(i - 1)$ and neuron k in layer i
- Layer weights are 2-d arrays.



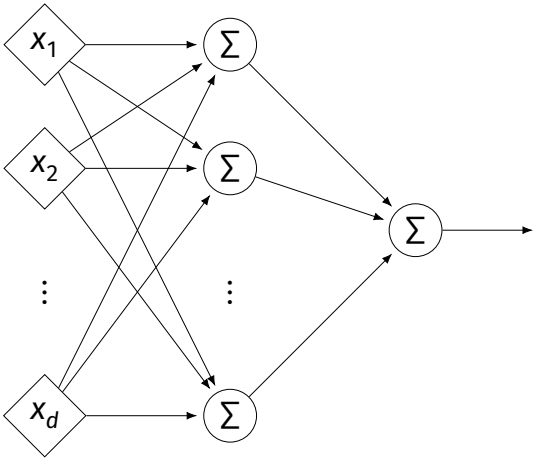
Notation

- Each hidden/output neuron gets a "dummy" input of 1.
- jth node in ith layer assigned a bias weight of $b_j^{(i)}$
- Biases for layer are a vector: $\vec{b}^{(i)}$

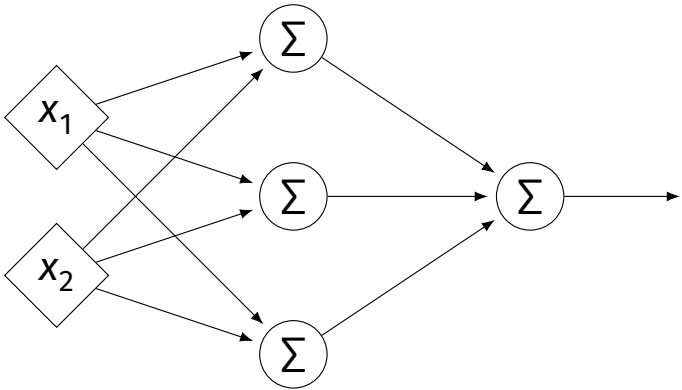


Notation

- Typically, we will not draw the weights.
- We will not draw the dummy input, too, but it is there.



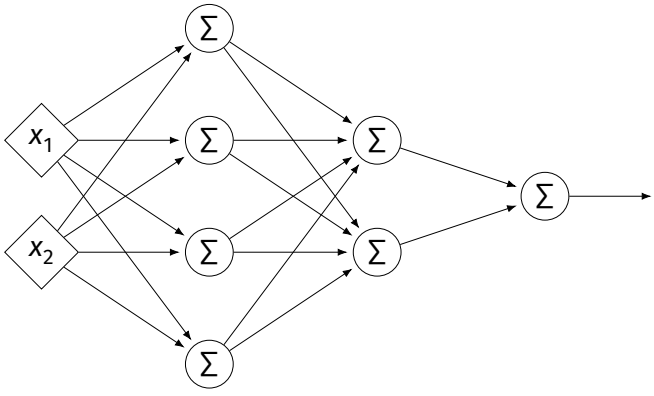
Example



$$W^{(1)} = \begin{pmatrix} 2 & -1 & 0 \\ 4 & 5 & 2 \end{pmatrix} \quad W^{(2)} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$

$$\vec{b}^{(1)} = (3, -2, -2)^T \quad \vec{b}^{(2)} = (-4)^T$$

Example



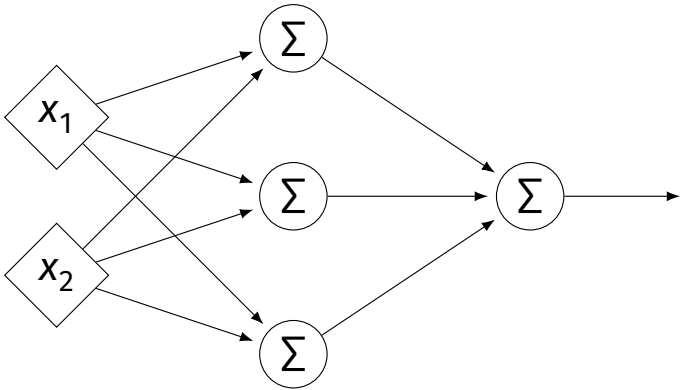
$$W^{(1)} = \begin{pmatrix} 2 & -1 & -3 & 0 \\ 4 & 5 & -7 & 2 \end{pmatrix} \quad W^{(2)} = \begin{pmatrix} 1 & 2 \\ -4 & 3 \\ -6 & -2 \\ 3 & 4 \end{pmatrix} \quad W^{(3)} = \begin{pmatrix} -1 & 5 \end{pmatrix}$$

$$\vec{b}^{(1)} = (3, 6, -2, -2)^T \quad \vec{b}^{(2)} = (-4, 0)^T \quad \vec{b}^{(3)} = (1)^T$$

Evaluation

- These are "**fully-connected, feed-forward**" networks with one output.
- They are functions $H(\vec{x}) : \mathbb{R}^d \to \mathbb{R}^1$
- To evaluate $H(\vec{x})$, compute result of layer i, use as inputs for layer $i + 1$.

Example



- $\vec{x} = (3, -1)^T$
- $z_1^{(1)} =$
- $z_2^{(1)} =$
- $z_3^{(1)} =$
- $z_1^{(2)} =$

$W^{(1)} = \begin{pmatrix} 2 & -1 & 0 \\ 4 & 5 & 2 \end{pmatrix}$ $W^{(2)} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$ $\vec{b}^{(1)} = (3, -2, -2)^T$ $\vec{b}^{(2)} = (-4)^T$

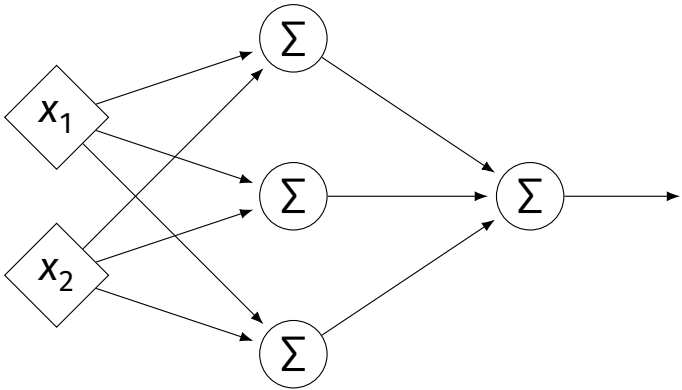
Exercise

What is the output of the network on the previous slide?

Evaluation as Matrix Multiplication

- Let $z_j^{(i)}$ be the output of node j in layer i.
- Make a vector of these outputs: $\vec{z}^{(i)} = (z_1^{(i)}, z_2^{(i)}, \ldots)^T$
- Observe that $\vec{z}^{(i)} = \left[W^{(i)}\right]^T \vec{z}^{(i-1)} + \vec{b}^{(i)}$

Example



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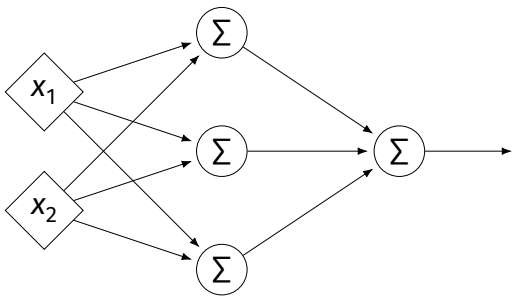
Each Layer is a Function

- We can think of each layer as a function mapping a vector to a vector.

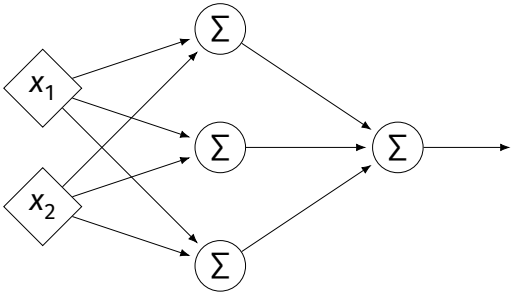
- $H^{(1)}(\vec{z}) = \left[W^{(1)}\right]^T \vec{z} + \vec{b}^{(1)}$
  - $H^{(1)} : \mathbb{R}^2 \to \mathbb{R}^3$

- $H^{(2)}(\vec{z}) = \left[W^{(2)}\right]^T \vec{z} + \vec{b}^{(2)}$
  - $H^{(2)} : \mathbb{R}^3 \to \mathbb{R}^1$



NNs as Function Composition

- The full NN is a composition of layer functions.



$$H(\vec{x}) = H^{(2)}(H^{(1)}(\vec{x})) = \left[W^{(2)}\right]^T \underbrace{\left(\left[W^{(1)}\right]^T \vec{x} + \vec{b}^{(1)}\right)}_{\vec{z}^{(1)}} + \vec{b}^{(2)}$$

NNs as Function Composition

- In general, if there k hidden layers:

$$H(\vec{x}) = H^{(k+1)}\left(\cdots H^{(3)}\left(H^{(2)}\left(H^{(1)}(\vec{x})\right)\right)\cdots\right)$$

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Representation Learning

Lecture 10 | Part 6

Activation Functions

The Power of NNs

- Our goal in connecting linear models together was to build a more powerful model.
- These neural networks must be more powerful than linear models, right?

The Power of NNs

- Our goal in connecting linear models together was to build a more powerful model.
- These neural networks must be more powerful than linear models, right?
- **Well, no**... not as they are currently defined.

Theorem

If $f(x)$ is a linear function, and $g(x)$ is a linear function, then $f(g(x))$ is again a linear function.

Result

- Our neural networks are just compositions of linear functions.
- The NNs we have seen so far are all equivalent to linear models!

$$H(\vec{x}) = \vec{w} \cdot \text{Aug}(\vec{x})$$

- For NNs to be more useful, we will need to add **non-linearity**.

Activations

- So far, the output of a neuron has been a linear function of its inputs:

$$w_0 + w_1 x_1 + w_2 x_2 + \ldots$$

- Can be arbitrarily large or small.

- But real neurons are **activated** non-linearly.
  - E.g., saturation.

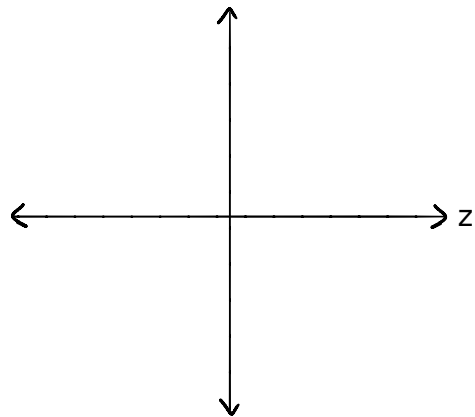
Idea

- To add nonlinearity, we will apply a non-linear **activation function** g to the output of **each** hidden neuron (and sometimes the output neuron).

Linear Activation

- The **linear** activation is what we've been using.

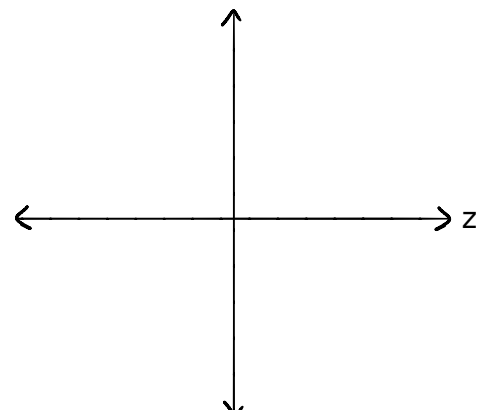
$\sigma(z) = z$



Sigmoid Activation

- The **sigmoid** models saturation in many natural processes.

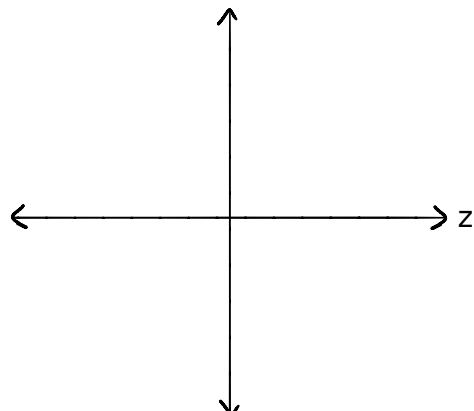
$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



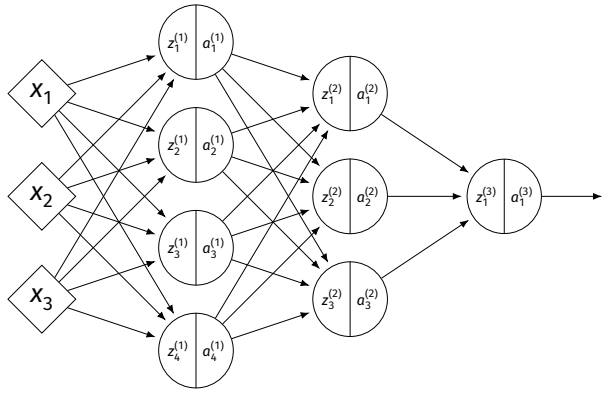
ReLU Activation

- The **Rectified Linear Unit (ReLU)** tends to work better in practice.

$g(z) = \max\{0, z\}$

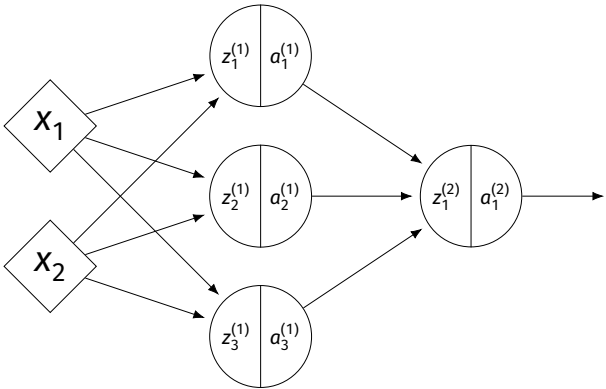


Notation



- $z_j^{(i)}$ is the linear activation before g is applied.
- $a_j^{(i)} = g(z^{(i)})$ is the actual output of the neuron.

Example



- g = ReLU
- Linear output
- $\vec{x} = (3, -1)^T$
- $z_1^{(1)} =$
- $a_1^{(1)} =$
- $z_2^{(1)} =$
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$$W^{(1)} = \begin{pmatrix} 2 & -1 & 0 \\ 4 & 5 & 2 \end{pmatrix} \quad W^{(2)} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} \quad \vec{b}^{(1)} = (3, -2, -2)^T \quad \vec{b}^{(2)} = (-4)^T$$

Output Activations

- The activation of the output neuron(s) can be different than the activation of the hidden neurons.
- In classification, **sigmoid** activation makes sense.
- In regression, **linear** activation makes sense.

Main Idea

A neural network with linear activations is a linear model. If non-linear activations are used, the model is made non-linear.